Exam Symmetry in Physics

Date	July 4, 2013
Room	X 5113.0201
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are not allowed to use the lecture notes, nor other notes or books
- The weights of the three exercises are given below
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Weighting

1a)	5	2a)	5	3a)	5
1b)	5	2b)	5	3b)	10
1c)	10	2c)	5	3c)	5
		2d)	5	3d)	5
		2e)	5		
		2f)	10		
		2g)	10		

$$\text{Result} \ = \frac{\sum \text{points}}{10} + 1$$

Exercise 1

(a) Write down the definitions of the groups Z_n, C_n, and the group consisting of the n distinct roots of unity, i.e. provide for each of the three groups the set of elements and the composition law.

(b) Provide the isomorphisms among these three groups and show that they are indeed isomorphisms.

(c) Provide a proof of Lagrange's theorem that states that the order $[h]$ of any subgroup of a finite group G is a divisor of the order $[g]$ of G.

Exercise 2

Consider the cyclic group C_4: gp$\{c\}$ with $c^4 = e$.

(a) Determine the order of the elements and the conjugacy classes of C_4.

(b) Construct the character table of C_4.

(c) Construct the three-dimensional vector representation D^V of C_4.

(d) Decompose D^V into irreps and use the result to conclude whether a crystal with C_4 symmetry can support a permanent electric dipole moment.

(e) Determine the Clebsch-Gordan series of the direct product representation $D^V \otimes D^V$ of C_4.

(f) Explicitly determine the tensors T^{ij} $(i, j = 1, 2, 3)$ that are invariant under C_4 and check whether the answer is in agreement with the result obtained in part (e) of this exercise.

(g) Find how the 3-fold degeneracy of an $l = 1$ atomic orbital is broken when the $SO(3)$ symmetry is reduced by a crystal environment to C_4.

Exercise 3

(a) Give the definition of an isometry in three dimensions.

(b) Show whether the orthogonal group $O(3)$ is an invariant subgroup of the Euclidean group $E(3)$. Hint: write all elements of $E(3)$ as $(O|\vec{a})$ and use that $(O_1|\vec{a}_1)(O_2|\vec{a}_2) = (O_1O_2|O_1\vec{a}_2 + \vec{a}_1)$.

(c) Show that the rotations around a fixed axis in three dimensions form an Abelian subgroup of $SO(3)$ that it is isomorphic to $SO(2)$.

(d) Demonstrate whether the defining representation of $SO(3)$ provides an irrep of the Abelian subgroup of $SO(3)$ discussed in part (c) of this exercise.